University of California, Berkeley
Physics 110A Spring 2003 Section 2 (*Strovink*)

MIDTERM EXAMINATION II

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for two $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. A photocopy of the four inside covers of Griffiths is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (25 points)
An idealization of the *Bainbridge mass spectrograph* consists of the following arrangement. The entire apparatus is immersed in a uniform constant magnetic field $\vec{B} = |B_0|\hat{z}$. In the semi-infinite volume $x < 0$, there is also a uniform electric field $\vec{E} = |E_0|\hat{y}$. At the point $(x = -|x_0|, y = 0, z = 0)$, a beam of nonrelativistic ions of positive charge q exists; there the ions have velocities that point exactly in the $+\hat{x}$ direction. At $x = 0$, a thin slit admits only those ions which still have $y = 0$. Thereafter, in the semi-infinite volume $x > 0$, the ions move under the influence only of the uniform magnetic field.

An ion detector is placed at $(x = 0, y = -|y_0|)$. If an ion is detected, what mass M does it have?

Problem 2. (35 points)
A cylindrical rod of radius b and length $L \gg b$ is centered on the origin and coaxial with the z axis. It has "frozen-in" magnetization $\vec{M} = |M_0|\hat{z}$.

(**a**) (15 points) To lowest order in b/L, what is the magnetic flux Φ_B through the rod in the $z = 0$ plane?

(**b**) (20 points) The flux Φ_B is returned through free space. Draw a circle centered on the origin in the $z = 0$ plane, of radius $s \gg L$ chosen so that 99% of the returned flux passes through the area bounded by the circle. To lowest order in b/L and L/s, what is s?

Problem 3. (40 points)
Consider a "flux tube" consisting of a thin cylindrical rod of radius a, composed of linear material with constant magnetic permeability $\mu/\mu_0 \equiv \mu_r \gg 1$. The rod is finely wound with n turns/m of ideally conducting wire. It is bent into a closed circle of radius $b \gg a$ to form a thin toroid that is centered at the origin with $\hat{z}$ as its axis of rotational symmetry.

(**a**) (15 points) What magnitude $|\mathcal{E}|$ of EMF must be applied to the wire circuit in order to cause the current $I(t)$ flowing in the wire to increase at the rate $dI/dt = \alpha$?

(**b**) (10 points) Write the Biot-Savart law (relating $d\vec{B}$ to $I\,d\vec{l}$). Then, taking advantage of the duality between the equations

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{and}$$
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \,,$$

write a Biot-Savart-like law, true in the limit that a is negligible, that relates $d\vec{E}$ to

$$\frac{d\Phi_B}{dt}\, d\vec{l} \,,$$

where Φ_B is the magnetic flux carried by the flux tube and $d\vec{l}$ is directed along the tube's axis.

(**c**) (15 points) For the conditions of part (**a**), using the result of part (**b**), find the electric field that is induced at the origin.